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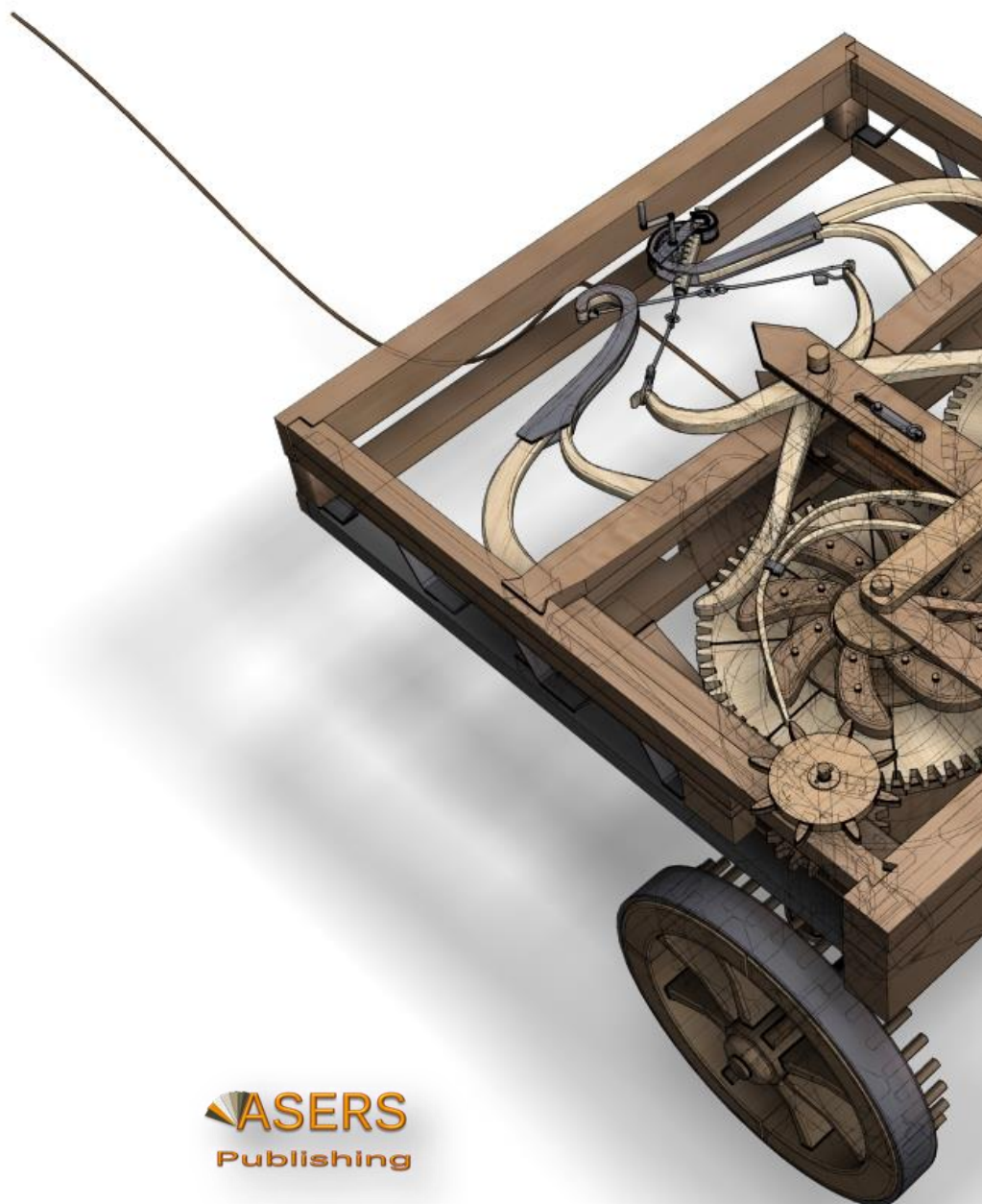Publishing

Contents:

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OVERSUPPLY OF LABOR AND OTHER PECULIARITIES OF ARTS LABOR MARKET

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Abstract:

There are several striking peculiarities of the arts labor market that have attracted the attentions of researchers in the last several decades: first is constant long run excess supply of arts labor; second, artists are more likely to be multiple-job-holders than other professions; third, artists pay significant earning penalties; finally, there are huge variations in artists earnings and huge inequalities among artists themselves. In order to explain these peculiarities, in this paper we developed two dynamic models of an artist's behavior and arts labor supply.

In the first model proposed here an artist is depicted as someone who is hired on the arts labor market and paid for his artistic time. In the second model an artist is described as someone who sells his products, like paintings for instance, on the market for artistic products. In order to make these models dynamic, an artist's productivity is here supposed to be a function of accumulated human capital of the artist. As a consequence, the supply of labor in the arts market appears as the result of an inter-temporal process of resources allocation. Both models end with the same result: the cost of producing a unit of an artistic commodity in a particular year should be equal to the present value of expected streams of all monetary and nonmonetary benefits generated by production of a given artistic unit. This result appears to be pretty suitable for formalization of several existing hypotheses aimed at explaining arts labor market peculiarities. Especially, by referring to the stream of expected nonmonetary benefits, models developed here are able to formalize the most promising among these hypotheses according to which an artist's need for self-discovery and self-actualization is the driving force in explaining the oversupply of arts labor.

Keywords: household production function, allocation of time, arts, expected benefits.

JEL Classification: Z10, Z11, J22, J24, J31.

1. Introduction

There are several striking peculiarities of the arts labor market that have attracted the attentions of researchers in the last several decades[29]. First and most important is constant oversupply of arts labor and related constant unemployment on the arts labor market. These phenomena can be documented with the plenty of data and anecdotal evidences since the end of nineteen century till our times. Persistent unemployment is not characteristic of other professions which might be characterized only with cyclical and temporary unemployment. Secondly, as a consequence, artists are more likely to be multiple-job-holders than any other professions. Very often, especially in the early years of their career, artists cannot cover even their basic needs by the earning from artistic work. Instead, they make living by doing non-artistic jobs. Thirdly, artists pay significant earning penalties. In other words, they have a lower level of average earnings than occupations with similar level of education. Interestingly enough, their earning from artistic job is not related to their formal artistic education. Even more interesting, their earning from non-artistic job is significantly related to their artistic education. Finally,

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[29] For more detailed survey and analysis of arts market peculiarities see Alper and Wassall (2006), Towse (2006), Menger (2006) and Menger (2002).

there are huge variations in artists' earnings and huge inequalities among artists themselves. Art is, obviously, risky business.

The explanations offered so far are based on David C. Throsby's model (Throsby (1994b)) of an artist's behavior. It explains this phenomenon by the operation of the income effect in the choice between earned income and arts time. Arts time, as we know, provides satisfactions to artists. Throsby's model is essentially static. It does not consider inter-temporal aspects of the supply of labor. These aspects are especially important if one considers the supply of arts labor. Artists' wages and prices of their works (paintings, for example) are a function of accumulated human capital of artists. This accumulation of human capital, on the other hand, can result both from investment in formal education and from previous artistic practice and experience of artists. Previous practice and experience is, in the case of arts, according to empirical works and casual observation, much more important than investment in formal education.

Once the analysis is expanded to capture an artist's human capital accumulation, the supply of labor in the arts market appears as the result of an inter-temporal process of resources allocation that is based on accumulation of a human capital decision. This dynamic extension of the basic model allows some other, even more important, peculiarities of the arts labor market to be explained (oversupply of arts labor, earning penalties, poverty among artists, and similar). In this article two such dynamic models of an artist's behavior have been developed. Both are based on a household production function approach. Both models are also based on the assumption that artists are multiple-job-holders and that they have to decide how much of their time to devote to artistic work and how much time to non-artistic work. It is in accordance with a casual observation that artists, especially in the early ages of their career, do both non-artistic jobs as well as artistic ones. The first model that is proposed here is based on the assumption that artists are hired on the arts labor market and paid for their artistic time. This approach was once proposed by Caserta and Cuccia (2001) but has not been solved and developed further. The second model is based on the assumption that artists sell their products, like paintings for instance, on the market. Labor supply is in this case derived from the artist's product supply function.

In the second section of the article a short survey of the static model developed by Throsby (1994b) is given first. For the sake of simplicity and comparability the model is a bit modified. The following two sections are the core of the article: in the third section the first dynamic model is given, while in the fourth section of the article the second model is presented. Implications of the models are discussed in the fifth section.

2. A Static approach to arts labor supply

In Throsby's work preference model (Throsby (1994b)) it is assumed that an artist maximizes the following utility function

$$U = U(L^M, X) \tag{1}$$

With L^M we present time devoted to an artistic activity, which by definition provides pleasure to the artist[30]. On the other hand, X presents quantity of all other market goods. Needless to say, both partial derivatives of this utility function are positive.

Artists are, of course, paid for their time devoted to art. If their hourly wage from this activity is w^M than their income earned from arts is equal to $L^M w^M$. Artists, however, have an option to devote part of their time to non-artistic activities (L^n). If their wage rate earned at a non-artistic job is w^n, than

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[30] Although in his formal analysis Throsby uses general utility function, in the graphical presentation he, in fact, applies quite specific kind of quasi linear utility function. By doing so he was able to present a typical artist as someone who has an absolute preference to artistic work once his basic needs are satisfied.

their income earned from non-artistic activities will be $L^n w^n$. An artist total income earned from both activities will be $L^n w^n + L^M w^M$. So, their income constraint becomes

$$pX = L^n w^n + L^M w^M$$

where: p presents the price of market goods. The crucial assumption of Throsby's work preference model is that non-arts wages are higher than arts wages, $w^n > w^M$.

Artists are also constrained by disposable time: the time they devote to artistic (L^M) and non-artistic (L^n) activities should be equal to their disposable time (L). Formally [31]

$$L^M + L^n = L$$

The time constraint and income constraint can be combined to give one constraint of the following form[32]

$$L^M w^n + pX = Lw^n + L^M w^M \qquad (2)$$

Behavior of an artist can now be outlined with expression (1) and (2). The artist chooses the value of L^M and X in order to maximize (1) subject to the constraint given by (2). In order to solve the problem we form Lagrange of the following form

$$\mathrm{L} = U(L^M, X) - \lambda[L^M w^n + pX - Lw^n + L^M w^M] \qquad (3)$$

The first order condition requires a partial derivative of with respect to L^M and X to be equal to zero. The second order condition will be, for the sake of simplicity, skipped. Solutions we get are[33]

$$\frac{\partial U}{\partial L^M} = \lambda(w^n - w^M)$$

$$\frac{\partial U}{\partial X} = \lambda p$$

Consequently, the optimal solution requires the marginal rate of substitution of artistic time for market goods (MRS) to be equal to

$$MRS = -\frac{\Delta X}{\Delta L^M} = \frac{\dfrac{\partial U}{\partial L^M}}{\dfrac{\partial U}{\partial X}} = \frac{w^n - w^M}{p}$$

$$\qquad (4)$$

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[31] In this presentation Throsbys' initial model is somewhat modified. We use the amount of disposable time devoted to artistic and non-artistic work while Throsby uses their share in disposable time.

[32] From the expression for time constraint it follows that $L^n = L - L^M$. By substituting this for L^n in income constraint and rearranging we get the constraint given by expression (2).

[33] Solution follows from the following two first order conditions:

$$\frac{\partial \mathrm{L}}{\partial L^M} = \frac{\partial U}{\partial L^M} + \lambda(w^n - w^M) = 0$$

$$\frac{\partial \mathrm{L}}{\partial X} = \frac{\partial U}{\partial X} - \lambda p = 0$$

Obviously, any time wage differential $w^n - w^M$ changes, either because of the change of w^n or because of the change of w^M, it will generate a substitution as well as an income effect. The substitution effect implies that when the wage differential is reduced more time will be devoted to a preferred artistic activity. It happens because, as expression (4) shows, shifting labor from non-pleasurable to pleasurable artistic activity is now less costly. It may happen for example as a result of the increase of wage rate of an artistic activity w^M. It is, indeed, something that happens during the artist's career as a result of his professional development. If, on the other hand, the wage differential increases, as a result of the increase of w^n for example, labor will be shifted from pleasurable to non-pleasurable activities: it is now more costly to shift labor from a non-pleasurable to pleasurable activity. This effect may be weakened or even reversed by the income effect. It may easily happen that the increase in wage differential caused by the increase of w^n results in shifting labor from non-pleasurable to pleasurable activities. In such a case higher income, resulting from the increase of wage rate of non-pleasurable activities, is used to "buy" time for pleasurable artistic activities. This is exactly something that characterizes the artist's behavior according to Throsby.

Apart from the substitution and income effect there is the price effect as well. Any increase of a market goods price will reduce the right hand side of expression (4). Conversely, any decrease of the price of market goods will increase it. This will produce a substitution as well as the income effect. If the price level increases, assuming overall income does not change, more time will be devoted to pleasurable activities: the cost of substituting non-pleasurable for pleasurable activities is now smaller. Conversely, if the price level decreases, assuming constant income, this cost will be higher and, as a consequence, more time will be devoted to non-pleasurable than to pleasurable activities. Note, however, that if the price level decreases and, as a consequence, income increases, this will put in force the income effect that may result in the increase of pleasurable relative to non-pleasurable activities.

The above is even more obvious if we watch the behavior of different labor shares in disposable time. Once we have the solution of the model (X^n) it is easy to calculate a share of artistic time in overall disposable time. Substituting solutions in equation (2), dividing it with L and transforming we get

$$l^M = \frac{w^n - px^*}{w^n - w^M} \tag{5}$$

where $l^M = \dfrac{L^M}{L}$, *while* $x^* = \dfrac{X^*}{L}$. This equation has the following partials, which describe the responsiveness of artistic labor supply to changes in commodity price, arts wage, and non-arts wage:

$$\frac{\partial l^M}{\partial p} = \frac{-x^*}{w^n - w^M} < 0$$

$$\frac{\partial l^M}{\partial w^M} = \frac{w^n - px^*}{(w^n - w^M)^2} > 0 \tag{6}$$

$$\frac{\partial l^M}{\partial w^n} = \frac{px^* - w^M}{(w^n - w^M)^2} < 0$$

Again, as we see, the model depicts artists as addicted to artistic work. Responsiveness to change in price in equilibrium is consistent with artists' peculiar behavior. Firstly, the more severe the budget constraints are, the less time artists will devote to artistic activities (the first partial). Secondly, the higher the arts wage is, the more time an artist will devote to artistic activities (the second partial). Finally and most interestingly, the higher the non-arts wage is, the more hours an artists will devote to their artistic activities (the third partial). As Merijn and Madden (2000) noticed, the model is less

spectacular for those artists whose arts wages are higher than the non-arts wage, $w^M > w^n$, and who, therefore, performs an artistic activity only. In that case all inequalities in expression (6) turn to zero.

Let us now see what happens if we assume that artists sell their artistic products instead of their artistic time. Assume that quantity of artistic products is, in that case, determined by the artist's production function of the following form

$$M = M(L^M, X^M)$$

where X^M stands for quantity of market goods purchased for the production of artistic products (raw material). If prices of artistic products are given by p^A and prices of raw material s are given by p^M, then the constraint given in expression (2) becomes

$$L^M w^n + pX + p^M X^M = Lw^n + M(L^M, X^M)p^A \tag{7}$$

Accordingly, new Lagrange gets the following form

$$\mathrm{L} = U(L^M, X) - \lambda[L^M w^n + pX + p^M X^M - Lw^n - M(L^M, X^M)p^A] \tag{8}$$

The solution of this problem is

$$\frac{\partial U}{\partial L^M} = \lambda\left(w^n - \frac{\partial M}{\partial L^M}p^A\right)$$

$$\frac{\partial U}{\partial X} = \lambda p$$

$$\frac{\partial M}{\partial X^M}p^A = p^M$$

Since $\dfrac{\partial M}{\partial L^M}$ presents a marginal product of labor engaged at artistic activities it follows that $\dfrac{\partial M}{\partial L^M}p^A$ presents the value of a marginal product of labor engaged at artistic activities. In the competitive market this should be equal to an arts gross wage, w^M. Obviously, this approach gives the same solution as the previous one.

Recently Casacuberta and Gandelman (2012) developed more general and more sophisticated model of arts labor supply than one developed by Thorsby and presented above. Firstly, their treatment of leisure is more general. Secondly, they do not assume that artists have absolute preferences toward artistic practicing. Finally, they introduce nonnegative constraints on artistic and non-artistic time and, in that way, explicitly allow for possibility of corner solution. The most general results of this model are, however, close to those proposed by Throsby.

Both above mentioned models are, however, static in its nature. This means that, as Caserta and Cuccia (2001) noticed, an "artist has no past and no future and that wage differential is entirely exogenous". In what follows two dynamic models of an artist's behavior will be developed. The past and future of the artist will be incorporated in it, while wage rates and prices of his products will be endogenously determined.

3. A dynamic model with arts labor hired

In order to develop a sophisticated dynamic model of artist behavior we will rely on household production function approach and the theory of allocation of time developed by Gary Becker and his

followers. Especially important here are papers and works written by Becker (1965), Becker (1991), Becker (1993), Grossman (1971), Ghez and Becker (1975), Michael and Becker (1973), Stigler and Becker (1977), Becker *et al.* (1991). According to this approach consumers run the production process using market goods, their own time and other inputs in order to produce commodities for the final consumption. "These commodities include children, prestige and esteem, health, altruism, envy, and pleasure of the senses, and are much smaller in number than the goods consumed" as Becker noticed once (Becker (1991), p. 24). A meal, for example, according to this approach should be understood as a commodity produced using goods purchased, our own time used for purchase of goods and cooking, and the ability to cook as a kind of human capital. Similarly, appreciation of music, as a kind of commodity of "the pleasure of senses", is made by combining market goods or services (CDs, instruments, concerts, music lessons), our own time devoted to it, and the ability to appreciate music, which again depends on specific human capital of individuals. Consequently, our decisions about consumption of certain commodities are governed not solely by market prices of goods and services used in producing certain commodities but by shadow prices of commodities, which also includes the opportunistic price of our time, the price of human capital, and prices of all other household resources involved in production of a given commodity. Therefore, changes in the pattern of demand of market goods and services may not necessarily be the result of changes in market prices and in our tastes but rather the result of changes in the household production technology and / or in the inputs available for production.

More precisely, consumers are, according to this approach, supposed to maximize their utility function subject to the money income constraint, time constraint and to a household production function constraint (Becker (1965), Michael and Becker (1973)). Note that traditional theory of consumer behavior takes into account only the money income constraint. As a consequence the traditional approach gives as a result only the allocation of household money income among different goods and services. The solution of the new approach, on the other hand, apart from the allocation of earnings among different goods, provides the allocation of time among work and consumption as well as allocation of time among different kinds of consumption. Consequently, changes in shadow prices of commodities, which are governed by changes in market goods prices and wage rates, will give rise to different allocation of time and different allocation of money income of households. Note, however, that changes in allocations of earnings and time may result not only from changes in market prices and wages but also from the previous consumption history of individuals. Consumption of certain commodities and experience gained in that way may, in fact, result in the increase of human capital that is relevant for future production of a given commodity. In this case, the cost of production and the shadow price of this commodity will be reduced as a result of this human capital accumulation. Consequently, the consumption of the commodity whose shadow prices falls relatively to others will be increased. This effect is now well known as an addictive effect.

Stigler and Becker (1977), who first discussed addiction effect, choose music appreciation as an example of such an addictive commodity. According to this interpretation, consumption of music is never simple consumption of market goods or services, but rather the consumption of output of productive process that combines market goods and services, consumers' time, human capital and other inputs. Human capital expressed as the ability to enjoy music is of crucial importance. It is an increasing function of weighted cumulative of the previous consumption of music. The more time someone devotes to music consumption, the more knowledgeable and perceptive he becomes, and in that way more productive he will be the next time he consumes music. This effect will reduce the shadow price of music consumption and in that way make music more attractive relative to other commodities. Increased consumption of music will contribute to further increase of human capital, which in turn will further decrease the shadow price and increase consumption of music. And so on. The same applies for all other kinds of art appreciation: having artistic paintings or visiting artistic galleries and museums, watching dramas or operas in theatres, enjoying movies at cinemas or at homes by using CDs, and similar.

Artists, on the other hand, are prone to the same kind of addictive behavior towards art, but they are also paid for the time they devote to the artistic practicing. They "enjoy" the time they devote to practicing art. In other words, they produce and consume the commodity known as art. The quantity of this commodity in a year j of an artist's career will be presented by M_j. They also produce and consume all other commodities. In order to make things simple we will assume that quantity of all other commodities in a year j can be presented as the commodity Z_j. In that case, the utility function which is being maximized by the artist's household is given by

$$Uf = \sum_{j=1}^{n} \left(\frac{1}{1+\rho}\right)^j U\left[M_j, Z_j\right] \tag{9}$$

where ρ stands for the time preference rate, while n presents the remaining years of a career of the particular artist.

In order to produce a commodity of art artists use market goods, their time, and human capital relevant for production of this commodity. This can formally be presented using the following artists' household production function

$$M_j = M\left[X_j^M, L_j^M, H_j\right] \tag{10}$$

where X_j^M presents quantity of market goods used in producing arts in a year j, L_j^M stands for time devoted to arts production in a year j, while H_j stands for human capital used for that purpose in a year j. M_j is an output of this production function, but it is also an argument in the utility function. This output will be from now on called simply art and will be expressed in some kind of an efficiency unit. Each unit of time devoted to art will produce the same amount of art efficiency units as long as the amount of human capital stays the same. When the amount of human capital changes, the number of art efficiency units per unit of time changes. Note that this household production function differs from the one used by Stigler and Becker (1977) to describe behavior of a consumer of arts because it explicitly uses market goods as an argument of the function. It also differs, in the same way, from the production function proposed by Caserta and Cuccia (2001) for the description of an artist's behavior. Although artistic market goods may be skipped when dealing with production function of arts consumers, it is pretty unrealistic to miss such an important input when dealing with production function of an artist.

Production of all other commodities can be presented with the household production function of the following form

$$Z_j = Z\left[X_j^Z, L_j^Z\right] \tag{11}$$

Z_j stands for the quantity of all other commodities in a year j, X_j^Z presents quantity of market goods purchased for the production of all other commodities in a year j (purchase of food, shoes, clothing, and similar), while L_j^Z measures time used in production of these commodities in a year j (time to buy goods, time to make meals, to put make up, and similar).

Human capital, which is an argument in the production function of an artistic commodity, is itself the function of previous artistic experience and production of art. This is how addictive effect enters in our analysis. It can formally be presented by the following human capital production function

$$H_j = h\left[M_{j-1}, M_{j-2}, M_{j-3}, ..., E_j\right] \tag{12}$$

So, we assume that the entire work history of an artist can have influence on his human capital relevant for production of an artistic commodity. In order to allow for influence of formal education on artists' human capital we also introduce E_j as a measure of artists' years of education[34].

Important difference between artists and consumers of art is that artists are paid for their artistic experience. More experienced they are and more human capital they have, they will be able to produce more art efficiency units. So, hourly wages of artists will be the increasing function of their human capital. Formally

$$w_j^M = w^M\left(H_j\right) \tag{13}$$

In other words, an hourly wage of an artist in a year j is the increasing function of artists' human capital attained in that year, H_j. Obviously, their total earnings from artistic work in a year j are equal to $w_j^M L_j^M$. It is important to have in mind that by w_j^M we do not understand the wage per hour of time in which an artist is hired, but the wage per hour of the entire time devoted to arts practicing. As a result, changes of w_j^M are the result of simultaneous changes of a wage per hour of time in which an artist is hired for his artistic works and consequent changes of a share of time the artist is being hired in total time devoted to art practicing L_j^M.

Note, however, that artists unlike most of other professions and workers very often make their living by working other non-artistic jobs as well [35]. Income earned by artistic practice is most of the time, especially in the early ages of their career, not enough to support their living and their artistic persuasion. So, they devote L_j^n units of disposable time in a year j doing non-artistic jobs. Assuming for the sake of simplicity that the wage of non-artistic work is constant and equal to w^n, we conclude that their non-artistic income in a year j should be equal to $w^n L_j^n$. Their total earnings from artistic and non-artistic work are therefore equal to $w^n L_j^n + w_j^M L_j^M$. The above consideration is, of course, based on the assumption that hourly wages of non-artistic jobs are higher than those of artistic jobs. The reason why in this situation artists devote part of their disposable time to art lies, of course, in the fact that, apart from gaining certain money income from artistic production, artists receive significant stream of nonmonetary benefits that artistic practice brings by itself. If and when, in later years of their career, artists' wages approach the level of non-artistic wages or above it artists devote their entire working time

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[34] Existing researches, although scarce, seem to suggest three interesting conclusions regarding the importance of human capital for artists' productivity and for their earnings. Firstly, years of schooling, as a measure of formal education, have no influence on artists' earnings from artistic works. In some researches even negative coefficients have been obtained. Secondly, formal education has significant influence on artists' earnings from non-artistic works. Finally, artistic earnings are found to be influenced by years of artists' experience, which measure on-the-job-training or learning by doing as it is sometimes called. While the first two results are surprising, the last one is quite in line with findings for other professions. It is probably the reason why this result has not been stressed enough in previous researches. Nevertheless, it is very important and it motivates the approach taken in this paper. For more detailed survey and discussion of the above issues see Alper and Wassall (2006), Towse (2006), Throsby (1994b), Throsby (1994a), Throsby (2007) and Caserta and Cuccia (2001).

[35] Multiple job-holding is widespread among artists. It has been documented and analyzed by a great number of cultural economists. See for example: Throsby (1992), Throsby (1994b), Throsby (1994a), Throsby (2007), Jeffri Joan and Mary (1991), Merijn and Madden (2000), Alper and Wassall (2006) and Abbing (2004). A more subtle analysis would require, as Merijn and Madden (2000) and Throsby (2007) insisted, artistic working time to be divided into non-art, art related (like teacher of art, for example), and pure art working time. Within pure artistic working time it is also possible to make further distinction between those activities that are more artistically rewarding but less financially attractive and those activities that are less artistically rewarding but more financially attractive (Throsby (2007), Bryant and Throsby (2006)). No doubt, even pure artistic jobs differ among themselves by the quantity of artistically rewarding activities in their content. All this can make empirical analysis of this market much more complicated than in other cases.

to artistic jobs. More generally, the higher the level of w_j^M, the higher proportion of L_j^M in disposable time will be. The artist's career is characterized by a pretty stable increase in human capital and, therefore, by ever increasing value of w_j^M, which naturally leads to the increase of L_j^M.

Relying on previous considerations, we can now introduce two additional constraints encountered by artists. The first one is the income constraint and it says that total artist's income from work and from his wealth in a year should be equal to his market goods expenditure in the given year. More precisely and formally

$$\sum_{j=1}^{n}\left(\frac{1}{1+r}\right)^j \left[p^Z X_j^Z + p^M X_j^M \right] = \sum_{j=1}^{n}\left(\frac{1}{1+r}\right)^j \left[w^n L_j^n + w_j^M L_j^M + b_j \right] \tag{14}$$

where r stands for the interest rate, p^Z presents prices of market goods used for production of all other commodities, p^M presents prices of market goods used for production of an artistic commodity, while b_j stands for income earned from the artist's wealth and other sources (social assistance or artist's support programs, for example). Note also that, for the sake of simplicity, we assumed that prices of all market goods are constant in all considered years. Other important constraint that should be taken into account is the time constraint and it says that the time used for artistic and non-artistic work and the time used for production of all other commodities should be equal to the artist's disposable time in every year. Formally

$$L_j^n + L_j^M + L_j^Z + = L_j \tag{15}$$

where L_j presents the artist's disposable time in a year j. From (15) it follows that $L_j^n = L_j - L_j^Z - L_j^M$. By substituting this for L_j^n in income constraint (14) and by rearranging we get expression

$$\sum_{j=1}^{n}\left(\frac{1}{1+r}\right)^j \left[p^Z X_j^Z + p^M X_j^M + w^n L_j^Z + w^n L_j^M \right] = \sum_{j=1}^{n}\left(\frac{1}{1+r}\right)^j \left[w_j^M L_j^M + w^n L_j + b_j \right] \tag{16}$$

What we got here is, using Becker's terminology, "full income constraint" of an artist. The artist's decision making process is now simplified and presented by the utility function (9) which should be maximized under constraints given by the production functions (10) and (11) and the full income constraint (16). Of course, before that, human capital production function (12) should be substituted for H_j in expression (10), while expression (13) should be substituted for w_j^M in expression (16).[36] The problem can be further simplified by substituting values of X_j^Z, X_j^M, L_j^Z and L_j^M in expression (16) by values of these variables derived from household production functions (10) and (11). In that case the decision making process that outlines the artist's behavior can be described by expression (9) which should be maximized subject to a new modified constraint (16). In order to solve the problem we form the following Lagrange

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[36] To get even more realistic model of artist behavior, we could have introduced conditions of non-negativity of artistic and non-artistic time, which would, relying on Kuhn-Tucker, allow us to determine the switching point (corner solution): level of artistic job wage at which artist switch and devote all his disposable time to artistic job (or to non-artistic job). The only reason for not introducing these constraints is to avoid getting complicated and cumbersome results. On the other hand, in this particular case, switching point can be easily identified by simple intuition: when the sum artist's expected wages from artistic job and expected nonmonetary benefits from artistic job become equal to expected wages from non-artistic job, artist allocates his entire working time solely to artistic job.

$$L = \sum_{j=1}^{n} \left(\frac{1}{1+\rho}\right)^{j} U\left[M_j, Z_j\right]$$

$$-\lambda \sum_{j=1}^{n} \left(\frac{1}{1+r}\right)^{j} \left[p^Z X_j^Z + p^M X_j^M + w^n L_j^Z + w^n L_j^M - w_j^M L_j^M - w^n L_j - b_j\right] \qquad (17)$$

By the first order condition for Z_j, we get the following solution for a commodity Z_j (cf. Appendix 6.1)

$$\frac{\partial U}{\partial Z_j} = \lambda \left(\frac{1+\rho}{1+r}\right)^{j} \left[p^Z \frac{dX_j^Z}{dZ_j} + w^n \frac{dL_j^Z}{dZ_j}\right] = \lambda \left(\frac{1+\rho}{1+r}\right)^{j} \pi_{Z_j} \qquad (18)$$

where $\pi_{Z_j} = p^Z \dfrac{dX_j^Z}{dZ_j} + w^n \dfrac{dL_j^Z}{dZ_j}$ presents the shadow price of a commodity Z_j. This is a very known solution for allocation of time and income derived by Becker in his already quoted works. It simply says that the marginal utility of a commodity Z_j should be equal to marginal cost of all inputs involved in production of that commodity. Needless to say λ presents, as usual, the marginal utility of money income.

By the first order condition for M_j, we get the following solution for a commodity M_j (cf. Appendix 6.1)

$$\frac{\partial U}{\partial M_j} = \lambda \left(\frac{1+\rho}{1+r}\right)^{j} \left[\frac{p^M}{\dfrac{\partial M_j}{\partial X_j^M}} + \frac{w^n - w_j^M}{\dfrac{\partial M_j}{\partial L_j^M}}\right] - \lambda \sum_{v=j}^{n} \frac{(1+\rho)^j}{(1+r)^v} L_v^M \frac{dw_v^M}{dM_j}$$

$$-\sum_{v=j+1}^{n} \frac{(1+\rho)^j}{(1+r)^v} \frac{\partial U}{\partial M_v} \frac{\partial M_v}{\partial H_v} \frac{dH_v}{dM_j} \qquad (19)$$

By rearranging it we get an equally useful expression

$$\left[p^M \frac{\partial X_j^M}{\partial M_j} + w^n \frac{\partial L_j^M}{\partial M_j}\right] = \frac{1}{\lambda}\left(\frac{1+r}{1+\rho}\right)^{j} \frac{\partial U}{\partial M_j} + \frac{1}{\lambda} \sum_{v=j+1}^{n} \frac{(1+r)^j}{(1+\rho)^v} \frac{\partial U}{\partial M_v} \frac{\partial M_v}{\partial H_v} \frac{dH_v}{dM_j}$$

$$+ w_j^M \frac{\partial L_j^M}{\partial M_j} + \sum_{v=j}^{n} \frac{(1+r)^j}{(1+r)^v} L_v^M \frac{dw_v^M}{dM_j} \qquad (20)$$

Expression (20) provides some important results. The left hand side of this expression presents the value of all costs committed by an artist in a year j for the production of a unit of arts commodity. As we see, this is very similar to the shadow price π_{Z_j} obtained previously for the commodity Z_j in expression (18). So, we can say that it presents the shadow price of a unit of commodity M_j produced in a year j. On the right hand side we have four parts. Together, all of them present a stream of benefits that an artist has from this arts unit production in a year j. The first two elements present monetary equivalent of nonmonetary benefits generated by producing a unit of arts, while the last two

elements present monetary benefits generated by this production. The first and third elements present benefits grasped immediately in a year j. The second and forth elements, on the other hand, present streams of benefits that are supposed to be generated from year j to the end of the artist's career, given as a present value in a year j. More precisely, the first element on the right hand side presents monetary equivalent of nonmonetary benefits gained in a year j which is the result of "pleasure" that dealing with art itself provides to artists. Since, however, the artist's activity in a year j increases his future human capital in all years that follow up to the end of her career $\left(\partial H_v / \partial M_j \right)$, it will inevitably contribute to the increase of productivity of his artistic production $\left(\partial M_v / \partial H_v \right)$ in all years that follow. This, in turn, will contribute to the increase of his future "pleasure" of dealing with art $\left(\partial U / \partial M_v \right)$ in all years that follow to the end of his career. This future stream of nonmonetary benefits is presented by the second part of the right hand side of expression (20). The third element is easy to understand: it presents the wage earned in a year j from producing a unit of art. However, monetary benefits do not end with this. Since the artist's activity in a year j contributes to the increase of human capital in all years that follows up to the end of the artist's career and since the artist's future wages are influenced by this increase of human capital $\left(\partial w_v^M / \partial M_j \right)$, we can expect the artist's activity in a year j to produce a stream of wage increase in all years that follows up to the end of the artist's career. This stream of future monetary benefits is given by the last part of expression (20). At the end of this section, it is worth noting that results given in expressions (19) and (20) are, in fact, very simple and in accordance to what someone would expect to get by relying on simple intuition.

4. A Dynamic model with arts products sold

In the previous consideration we assumed that artists are paid for the time they devote to art practicing. In many cases this is a pretty realistic picture of what is really happening on the market. Actors, singers, musicians, dancers and other artists engaged in so called *performing arts* are, for example, paid for their time being hired. In that case artistic organizations that hire them have their own production function and their own (profit, artistic quality or other) maximizing goals. Demands for artists' labor and other inputs are in that case derived from this process of maximization under production function and other constraints encountered by these organizations. Other kinds of artists are, however, paid for their products, that is for what we notified previously with M_j. A creative painting is an obvious example[37]. Creative painters are paid for their pictures. The same apply for sculptors, writers, composers, craftspeople and some other kinds of so called *creative artists* [38]. The decision making process is in this case somewhat different. Artists are paid for their products. Their household production function directly generates the production function of the entire art industry. The labor supply

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[37] In the case of creative painting it is necessary to make distinction between primary and secondary market of creative paintings. Our focus here is on the primary market of paintings and on prices of paintings on that market. For more detailed exposition of the primary and secondary market of creative paintings and of artistic works market in general see Heilbrun and Gray (2004), p. 165-187.

[38] Although somewhat blurred, the distinction between creative and performing artists is a very useful one. Merijn and Madden (2000) pointed out to seven important differences between them. The first one is that creative artists are self-employed, while performing artists work under short time contracts. The second one, and for this analysis the most important one, is related to the previous one: creative artists are paid for their "products", while performing artists are paid per hour hired. The third one is that creators are restricted by income constraint, while performers mostly have restriction regarding availability of works and contracts. The forth one is that creators work individually, while performers work with others. The fifth one is that the work of creators is valued according to their innovations, while the work of performers is characterized by craftsmanship and technical skill. The sixth one is that creators have high production costs, while performers have low production costs. Finally, creators are not unionized like performers.

function is derived from the process of maximization under these constraints. In what follows we will try to describe the artist's behavior in this very common situation.

As in the previous case artists are supposed to maximize the utility function (9). Household production functions are also given by previous expressions (10) and (11). The human capital production function, which is supposed to be substituted in (10), is like before given with expression (12). The time constraint that an artist encounters is also the same, expression (15). Their income constraint is, however, different from that in the previous case. Of course, they are, for the same reason as before, supposed to do artistic as well as non-artistic jobs. Their earnings from non-artistic jobs are the same as in the previous case and they are given by $w_n L_j^n$. However, since they are selling their artistic products, their earnings from artistic work are now given by the following expression:

$$p^A M_j = p^A M \left[X_j^M, L_j^M, H_j \right]$$

A new element here is p^A and it presents the price of an artistic product measured per efficiency unit. We supposed here that this market price is constant. Note, however, that this does not mean that the price of the artist's works (paintings, for example) does not change during the artist's career. On the contrary, prices of the artist's works will increase as a result of accumulation of human capital, which is given as a function of previous artistic experience of the artist. To understand this notice that, as we already said, M_j does not measure quantity of works but quantity of works of the same efficiency units. It measures not just the number of creative products (paintings, for example), but their quality as well. And the quality is what increases as a result of human capital accumulation. We may, for example, measure it in efficiency units of an artist with no experience, that is, with zero years of experience. The number of creative works made by an artist with j years of experience can be in that case presented by $M \left[X_j^M, L_j^M, H_0 \right]$. The previous expression for an artist's earnings from artistic work can be transformed in the following way:

$$p^A M_j = p^A \left(\frac{M \left[X_j^M, L_j^M, H_j \right]}{M \left[X_j^M, L_j^M, H_0 \right]} \right) M \left[X_j^M, L_j^M, H_0 \right] = p_j^a M \left[X_j^M, L_j^M, H_0 \right]$$

Note that:

$$p_j^a = p^A \left(\frac{M \left[X_j^M, L_j^M, H_j \right]}{M \left[X_j^M, L_j^M, H_0 \right]} \right) \tag{21}$$

presents price of one painting or one artistic work in general of an artist with j years of practice, while:

$$\frac{M \left[X_j^M, L_j^M, H_j \right]}{M \left[X_j^M, L_j^M, H_0 \right]}$$

stands for the number of efficiency units per one painting / work of an artist with j years of practice. More precisely it presents quality of the painting measured in efficiency units of the artist's first picture in his career. It is obvious, from expression (21), that the prices of the artist's products are not

constant and that they have their own time path during the artist's career. They are a function of an artist's human capital and they increase with the artist's years of experience[39].

This picture is somewhat complicated by the fact that, especially now days with the development of new reproducing technologies, in very many cases artists do not sell their products, but rather sell their copy rights to publishing and recording companies. This is a case with writers, composers, some singers, and similar. In return they get a stream of income, known as a royalty (10 to 15% of earnings), rather than a lump sum of money in the form of the price of an artistic product p_j^a. In order to make the analysis simple, we will assume that in these cases artists also receive the price p_j^a for their products. This price will be defined here as equivalent to the net present value of the expected stream of royalties[40]. Bearing this clarification in mind we can now provide the following modified expression for the artist's income constraint:

$$\sum_{j=1}^{n}\left(\frac{1}{1+r}\right)^{j}\left[p^{Z}X_{j}^{Z}+p^{M}X_{j}^{M}\right]=\sum_{j=1}^{n}\left(\frac{1}{1+r}\right)^{j}\left[w^{n}L_{j}^{n}+p^{A}M_{j}+b_{j}\right] \tag{22}$$

From the time constraint it follows that $L_{j}^{n}=L_{j}-L_{j}^{Z}-L_{j}^{M}$. By substituting for L_{j}^{n} in income constraint (22) and rearranging, we get the full income constraint:

$$\sum_{j=1}^{n}\left(\frac{1}{1+r}\right)^{j}\left[p^{Z}X_{j}^{Z}+p^{M}X_{j}^{M}+w^{n}L_{j}^{Z}+w^{n}L_{j}^{M}\right]=\sum_{j=1}^{n}\left(\frac{1}{1+r}\right)^{j}\left[p^{A}M_{j}+w^{n}L_{j}+b_{j}\right] \tag{23}$$

Behavior of artists can now be outlined by the utility function (9) which is supposed to be maximized under constraints given by the production functions (10), (11) and the full income constraint (23). The problem can be further simplified by substituting values of $X_{j}^{Z}, X_{j}^{M}, L_{j}^{Z}$ and L_{j}^{M} in expression (23) by values of these variables derived from household production functions (10) and (11). The decision making process that describes the artist's behavior can now be described by the utility function (9) which should be maximized subject to a newly modified constraint (23). In order to solve the problem we can form the following Lagrange:

$$\mathrm{L}=\sum_{j=1}^{n}\left(\frac{1}{1+\rho}\right)^{j}U\left[M_{j},Z_{j}\right]$$

$$-\lambda\sum_{j=1}^{n}\left(\frac{1}{1+r}\right)^{j}\left[p^{Z}X_{j}^{Z}+p^{M}X_{j}^{M}+w^{n}L_{j}^{Z}+w^{n}L_{j}^{M}-p^{A}M_{j}-w^{n}L_{j}-b_{j}\right] \tag{24}$$

The first order condition for the solution of the problem requires that partial derivatives of Lagrange with respect to M_{j} and Z_{j} be equal to zero. For the sake of simplicity, we again skip consideration of the second order condition. Using the described procedure we get the solution for commodity Z_{j} which is exactly the same as the one obtained previously in expression (18). As we

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[39] Note, however, that apart from time path of painters an average price of a picture, every picture produced in the particular year has its' own time path. This time path is determined by the forces that determine movements on the secondary market of pictures (Heilbrun and Gray (2004)).

[40] For interesting discussion on the issue of human capital and copy rights see Towse (2006). Note also that in some countries even painters receive part of their income in the form of a stream of income. This is the case in all countries that have adopted the so called resale right (droit de suite) according to which authors receive percentage of price every time his picture is resold (3% in EU, 5% in California). For more detailed discussion see Heilbrun and Gray (2004), p. 176.

know, the marginal utility of commodity Z_j should be equal to the marginal cost of all inputs involved in production of that commodity, which is equal to the shadow price of commodity Z_j.

By the first order condition for M_j, we get the following solution for a commodity M_j (cf. Appendix 6.2)

$$\frac{\partial U}{\partial M_j} = \lambda \left(\frac{1+\rho}{1+r}\right)^j \left[\frac{p^M}{\frac{\partial M_j}{\partial X_j^M}} + \frac{w^n - p^A \frac{\partial M_j}{\partial L_j^M}}{\frac{\partial M_j}{\partial L_j^M}} \right]$$

$$-\lambda p^A \sum_{v=j+1}^{n} \frac{(1+\rho)^j}{(1+r)^v} \frac{\partial M_v}{\partial H_v} \frac{dH_v}{dM_j} - \sum_{v=j+1}^{n} \frac{(1+\rho)^j}{(1+r)^v} \frac{\partial U}{\partial M_v} \frac{\partial M_v}{\partial H_v} \frac{dH_v}{dM_j} \tag{25}$$

By rearranging it we get the following useful expression

$$\left[p^M \frac{\partial X_j^M}{\partial M_j} + w^n \frac{\partial L_j^M}{\partial M_j} \right] = \frac{1}{\lambda}\left(\frac{1+r}{1+\rho}\right)^j \frac{\partial U}{\partial M_j} + \frac{1}{\lambda} \sum_{v=j+1}^{n} \frac{(1+r)^j}{(1+\rho)^v} \frac{\partial U}{\partial M_v} \frac{\partial M_v}{\partial H_v} \frac{dH_v}{dM_j}$$

$$+ p^A + p^A \sum_{v=j+1}^{n} \frac{(1+r)^j}{(1+r)^v} \frac{\partial M_v}{\partial H_v} \frac{dH_v}{dM_j} \tag{26}$$

By comparing expressions (25) and (26) with previously derived expressions (19) and (20), we notice a striking similarity among them. Notice first that an element $p^A(\partial M_j / \partial L_j^M)$ in the first bracket in expression (25) presents the value of marginal product of labor, which increases with j and which is equal to the wage paid for artistic work. Formally,

$$p^A \frac{\partial M_j}{\partial L_j^M} = w_j^M$$

Bearing that in mind we conclude that the first part of expression (25) is equal to the first part of expression (19). It is intuitively clear, but it can be formally proved, that the same equality exists among the second part of expression (25) and the second part of expression (19). They present an expected value of increases of earnings from $j+1$ year to the end of the artist's career, caused by unit production in a year j and given in the present value in a year j. Finally, the third parts of compared equations are equal by definition, and they present expected streams of nonmonetary benefit increases that unit production in a year j generates to the end of the artist's career. Looking at expression (26) we see that its left hand side again presents the shadow price of a unit of commodity produced in a year j. The right hand side of this expression presents as before a stream of all benefits generated by a unit of commodity M_j up to the end of the artist's career. To conclude, expressions (25) and (26) have the same meaning as already discussed expressions (19) and (20) and their implication will be considered together in the next section.

5. Implications

As we already noticed, expressions (20) and (26), a stream of benefits generated by producing a commodity of art is much larger than that of ordinary goods, Z_j, implied by expression (18). As a consequence, artists will be motivated to allocate relatively much greater part of their resources in art

rather than in ordinary commodities, Z_j. The same can be seen by looking at expression (19) and (25) and comparing them with expression (18). As we see, expression (19) differs from expression (18) for ordinary commodities by three additional parts. Firstly, the value of labor resources used in production of art $\dfrac{w^n}{\partial M_j / \partial L_j^M}$ given in the bracket is reduced by the value of an artist's earnings from artistic work $\dfrac{w_j^M}{\partial M_j / \partial L_j^M}$. The similar effect is present in Throsby's model (Throsby (1994b)). Secondly, the shadow price of producing a commodity of art is further reduced by the value of a stream of increases of future wages caused by arts unit production in a year j, $\lambda \sum_{v=j+1}^{n} \dfrac{(1+\rho)^j}{(1+r)^v} L_v^M \dfrac{dw_v^M}{dM_j}$. Finally, it is also reduced by the value of a stream of increases of future nonmonetary benefits caused by arts unit production and consequent human capital creation in a year j, $\sum_{v=j+1}^{n} \dfrac{(1+\rho)^j}{(1+r)^v} \dfrac{\partial U}{\partial M_v} \dfrac{\partial M_v}{\partial H_v} \dfrac{dH_v}{dM_j}$. The last two effects are not present in Throsby's work preference model. Similar conclusions follow from examination of expression (25).

As a consequence, the marginal rate of substitution (MRS) between the commodity Z_j and the commodity M_j, given by

$$MRS = -\frac{\Delta Z_j}{\Delta M_j} = \frac{\dfrac{\partial U}{\partial M_j}}{\dfrac{\partial U}{\partial Z_j}}$$

$$= \frac{\lambda\left(\dfrac{1+\rho}{1+r}\right)^j \left[\dfrac{p^M}{\dfrac{\partial M_j}{\partial X_j^M}} + \dfrac{w^n - w_j^M}{\dfrac{\partial M_j}{\partial L_j^M}}\right] - \lambda \sum_{v=j+1}^{n} \dfrac{(1+\rho)^j}{(1+r)^v} L_v^M \dfrac{dw_v^M}{dM_j} - \sum_{v=j+1}^{n} \dfrac{(1+\rho)^j}{(1+\rho)^v} \dfrac{\partial U}{\partial M_v} \dfrac{\partial M_v}{\partial H_v} \dfrac{dH_v}{dM_j}}{\lambda\left(\dfrac{1+\rho}{1+r}\right)^j \left[p^Z \dfrac{dX_j^Z}{dZ_j} + w^n \dfrac{dL_j^Z}{dZ_j}\right]}$$

will be at the point of optimum much smaller than that of two ordinary goods. As a result, artists will devote much higher share of their resources to art than what will be the case if we suppose that M_j is another ordinary good. In fact, they will devote much more of their labor and other resources to arts than what Throsby's model (Throsby (1994b)) would suggest (see expression (4)). Two additional effects, not captured by Throsby's model, are responsible for it. The first one is the effect of expected stream of future monetary benefits (the second term in numerator). The second one is the effect of expected stream of future nonmonetary benefits (the third term in numerator). Although not present in Throsby's model, these effects are, in fact, in line with Throsby's sort of argument. They further strengthen the importance of nonmonetary benefits in explaining artists' pure market performance and earning penalties evidenced in numerous researches on artists' earnings. For more detailed elaboration and analysis of artists' market performances see contributions of Alper and Wassall (2006), Towse (2006), Menger (2006), Menger (2002) and Throsby (2007).

Note, however, that it is not easy to say what would be the time pattern of $\partial U / \partial M_j$ and MRS during the artist's career. There is no reason to believe that it will be ever decreasing by the passage of time, as one might be prompted to conclude. Human capital creation resulting from art practicing has, in fact, two contradicting course of influence on these two values. On the one hand, it has ever decreasing influence on the first part of expressions (19) and (25) given in the bracket. This decreasing effect is twofold. Firstly, by the increase of j it is natural to expect an increase of an artist's human capital and consequent increase of the artist's wage from an artistic job, w_j^M. This, in turn, will cause a decrease of the shadow price of a commodity M_j, that is a decrease of the first part of expressions (19) and (25). Secondly, the constant increase of human capital by the passage of time will increase both marginal productivity of arts labor $\left(\partial M_j / \partial L_j^M \right)$ and marginal productivity of arts market goods $\left(\partial M_j / \partial X_j^M \right)$. This, in turn, will contribute to further decrease of the first part of expressions (19) and (25). As a consequence, older artists are by passage of time motivated to devote more resources to an artistic commodity and art production. Specifically, owing to this effect at the certain point of time in their career artists devote their entire working time to the artistic jobs. Formally speaking L_j^n becomes equal to zero, artists stop doing two kinds of jobs, and their total earning becomes equal to $L_j^M w_j^M$. Needless to say, this is what determines switching point that we mentioned previously. There is plenty of empirical evidence to support this effect.

On the other hand, younger artists are also motivated to invest much of their resources in practicing of art. Their shadow prices of an art commodity are reduced by the second and third element of expressions (19) and (25). These two parts, as already said, present expected streams of monetary and nonmonetary benefits increases that practicing of arts in a year j brings to artist up to the end of their career. Quite naturally, the younger the artist the higher this effect is. In the case of very old and experienced artists this effect can vanish indeed. This effect explains why young artists have such a strong drive for their profession, even though their wages from artistic work are very low. It seems safe to say that young artists have almost absolute preferences toward practicing of art once their basic needs are satisfied. This effect explains why young artists usually do two jobs, artistic as well as non-artistic, and why a great number of young artists experience poverty during their career. Paradigmatic, in that sense, is the story reported by Abbing (2004) according to which different financial subsidies and other programs of Dutch government to reduce poverty among creative painters were not followed by the reduction of poverty as expected but by two unexpected results both supporting Throsby's idea that young artists have almost an absolute preference toward art practicing once their basic needs are satisfied. Firstly, the introduction of different subsidies was followed by the reduction of time young artists devote to non-artistic works. Secondly, the number of artists increased by the passage of time after the introduction of programs. Both phenomena can, obviously, be explained by the income effect. Surprising is, however, a sharp influence of this effect at such a low level of income.

So, we have two contradicting sets of factors that have an influence on movement of $\left(\partial U / \partial M_j \right)$ and MRS during an artist's career. One of them, the current arts wage and productivity of resources used in art production, are the result of artist's "*history*" and it has a decreasing influence on movement of the above variables. The second set of factors, expected monetary and nonmonetary benefits, present the artist's "*future*" and it has an increasing influence on their movement. The answer to the question which of these two effects is stronger is an empirical one. What we know in that respect is the fact that young artists devote much more of their working time to non-artistic jobs than older artists. Older artists, in fact, very often, at the certain point of time, stop doing non-artistic jobs. Needless to say, the answer to this question will be different for different kinds of arts and for different artists due to the specific characteristics and circumstances.

Several hypotheses have been offered so far in explaining oversupply of arts labor and other peculiarities of arts labor market discussed at the introductory section of this paper. Let us see where theoretical models developed here belong and how they match with existing hypotheses.

The first hypothesis, as we already know, is the one proposed by Throsby (1994b) according to which dealing with arts, apart from monetary benefits, brings a huge amount of nonmonetary benefits to artists. The models developed here, as already noticed, strengthen this argument even more by adding an effect of expected stream of future nonmonetary benefits as a motive for dealing with arts. The expected stream of utility derived from artistic practice can, of course, take the form of pure pleasure derived from artistic work as such, as Throsby insists, but can also take the form of excitement from expected recognition by peers and artistic public in general, and even more importantly the form of expected nonmonetary benefits derived from self-discovery and self-actualization.

The second hypothesis is the one proposed long ago by Adam Smith and advocated recently as a possible explanation by Towse (2006). According to this explanation young people tend to enter artistic labor market too frequently because they overestimate their talent and likelihood of their future success. In other words oversupply of artists is a result of a special kind of presbyopia (opposite of myopia) that is inherent to young people. Models offered in this paper also allow for such an explanation. Once we understand future monetary and nonmonetary benefits not as exact ones, but rather as expected ones, and once we allow that these expectations can be, for some reasons, systematically overestimated, the models proposed here offer a room for such reinterpretation as well. Of course, this explanation fits perfectly to behavioral economics kind of argument which insists that in modeling human behavior we should use agent's real "cognitive framework" and not one which assume that human beings are perfectly rational. If so, our model should be modified to incorporate this realistic "cognitive framework"[41]. Whether there are grounds for above mentioned systematic overestimations of expectations in the case of artists is very debatable question, however, as Alper and Wassall (2006) noticed.

The third hypothesis is offered by Santos (1976), who claims that artists belong to a class of risk-taking workers who are willing to trade off a small chance of huge financial rewards for a much larger chance of low earnings. By dealing with expected streams of earnings models offered in this paper can be used as a framework for this kind of thinking as well. Santos's hypothesis can explain high variations in earnings of artists, as well as excess supply of arts labor. However, it cannot explain earning penalties that characterize artists' earnings. Even more importantly, Santos does not explain why artistic occupations would attract such a disproportionate number of risk-takers.

The forth hypothesis can be interpreted as the one which insists that artistic occupations do not attract a disproportionate number of risk-takers but a disproportionate number of self-actualization-seekers. Self-actualization and self-discovery are characterized by permanent learning by doing and permanent search for innovations. This is not possible within routine activities that characterize ordinary jobs. Uncertainty is *sine qua non* for such kind of persuasion. As we know, there is plenty of uncertainty and plenty of possibilities for satisfaction of this motive in all kinds of artistic occupations. It is, therefore, neither presbyopia nor risk-seeking behavior that explains an artist's occupational choice, but rather a fact that uncertain artistic occupations offer plenty of possibilities for self-actualization. This idea, which also has long history - Aristotle, Hegel, Marx - has recently been advocated most prominently by Menger (2006). The models developed here fit perfectly with this important explanation: what we call expected stream of future non-monetary benefits refers mostly to artists' need for self-actualization and self-discovery.

Finally, the last hypothesis is the one that can be derived from the theory on the earnings of superstars (Rosen (1981), Adler (1985) and Adler (2006)). Unlike the second and third hypothesis this one is able to provide the explanation for earning penalties as well as for huge variations in artists' earnings and excess supply of arts labor. Superstars in arts, sports etc. are individuals who are able to attain enormous success in their profession and whose earnings are significantly greater than that of

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[41] For most recent and detailed survey and references on Behavioral Economics see Angner and Loewenstein (2012).

their competitors. Rosen (1981) claims this phenomenon to be the result of interaction of two factors, one on the demand side and the other on the supply arts side. On the demand side of the story there is a hierarchy of talent and preference of consumers to consume the most talented artist. On the supply side there is nearly perfect (costless) reproducibility of art (especially performing arts) that occurs as a result of technological advancement (CDs for example). In those circumstances every consumer is able to consume the best art, while the most talented artist is able to capture the whole market. This is known as a winner-take-all situation[42]. Adler (1985), Adler (2006) proved that existence of superstars cannot be explained solely by differences in talent. According to his explanation there are a lot of artists who poses a stardom-quality talent. What produces superstars is the need on the side of consumers to consume the same kind of arts that other consumers do. This need develops from the fact that consumption of art is not momentary experience but a dynamic process that is based on previous artistic experience and knowledge accumulated in that way. This accumulation of knowledge is, however, not a result of consumers own ability to learn and judge about an intrinsic value of artistic products. Rather, it is a result of a complex social interaction and social processes able to induce path dependency phenomenon in the consumption of art. Owing to Adler's amendments a theory of superstar becomes relevant not only for performing but also for creative arts as well. No doubt, the theory of superstar has a path-breaking importance in explaining peculiarities of arts labor market. It, however, does not contradict with the explanation offered here. On the contrary, two explanations are complementary. The stardom explanation is about the demand side of arts labor market. The explanation offered here is about the supply side of that market: it explains why some people indulge in such a peculiar market at all.

The models proposed in this paper can also be regarded as more general because they can describe not only behavior of artists but also behavior in all those cases where work itself brings pleasure to workers, as well as in all those cases where previous activities has influence on current shadow prices of commodities. Art is only a paradigmatic case in which these widespread phenomena are most obvious and easy to understand. The work of scientists is also a very obvious case although they rarely experience a poverty stage during their career as artists do. Consequently, non-scientific working time of scientists is equal to zero almost from the beginning of their career. Their time earning profiles can prove, however, that expected stream of monetary benefits, the last part of expression (20), plays an important role in explaining their behavior in the early ages of their career. More importantly, their readiness to accept much lower wage rates compared to those in consulting or R & D activities within companies can be easily explained by the fact that their stream of nonmonetary benefits, the first and second part of expression (20), is significant indeed. In fact, it is so significant that it becomes decisive for their decision to deal with science. No doubt, a lot of scientific results, which are crucial for the growth of our standard of living, are paid by the mere pleasure that scientists derive from their work. The same applies to journalists especially those dealing with investigative journalism. There are, no doubt, a lot of other professions that can and should be analyzed in a similar manner. What is more important, it seems that, as a result of technological advancement, the number of such professions is growing. Technological progress has dramatically increased, and it is expected to increase even more, demand for so called creative works. The models developed here can be used for the analysis of a creative worker's behavior in general.

Another interesting phenomenon that can be explained using the above models is nonpaid work of volunteers. In many cases volunteers' readiness to work for free can be simply explained by the stream of nonmonetary benefits that such engagements bring to them. In most of the cases, however, other reasons may be even more important. The work of volunteers is very often explained by the fact that working as volunteers in the field of your own profession, while living from the income earned by doing some other job can help you develop your profession and your CV to the level that can help you

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[42] One interesting empirical research testing winners-take-all hypothesis on DVDs market has been done recently by Walls (2009b). See also empirical researches of Mckenzie (2010), Nelson *et al.* (2007), Jozefowicz *et al.* (2008), Walls (2009a) and Pitt (2010).

get a position in your own preferred field of work. This in turn is supposed to increase your future monetary earnings as well as your future nonmonetary benefits.

Currently, volunteering work of professionals is not such a widespread phenomenon as it might become in the future according to some analysts. On the contrary, what we experienced in the last three decades within developed countries is a constant increase of wage premium paid to skilled labor (college graduates and above). This has occurred in spite of the fact that a share of skilled labor has increased dramatically. The most convincing explanation offered so far is the one according to which, due to skill-biased technological progress, demand for skilled workers has increased even faster than the supply of them (Krusell *et al.* 2000). If the supply of skilled labor continues to grow at the existing rate and if, due to creative-labor-biased technological progress that seems started by nineties, the demand for labor shifts more toward creative than simply skilled labor, we may easily find ourselves in the position of experiencing excess supply of skilled labor. In that case volunteering work among professionals might become widespread indeed. Volunteering may become an important screening mechanism for unveiling creative abilities of young professionals. In general, the whole market for professionals might take characteristics that are now regarded to be exclusive peculiarities of arts labor market.

Even more interestingly, technological progress is, at the same time, making all types of jobs easier to work. Galor and Weil (1993) developed the growth model that stylizes the facts that since the end of the sixties female participation in labor force and female wage rates have been growing relative to males. Those processes are explained by the fact that technological progress in the last half of the twentieth century reshaped requirements for almost all kinds of jobs by reducing dramatically "masculine" requirements and by increasing "brain" requirements, making, in that way, almost all jobs affordable to females and, consequently, increasing supply of labor in developed economies. To say that "masculine" requirements are reduced is somehow the same as to say that disutility of work is reduced. Disutility of work, on the other hand, is nothing but negative value of what we call pleasure from work. More formally it is a negative value of the first part of our expression (20) and (26) $\frac{1}{\lambda}\left(\frac{1+r}{1+\rho}\right)^{j}\frac{\partial U}{\partial M_{j}}$. By allowing this element to be negative as well as positive the models developed here become even more general and able to explain a much wider span of economic and social phenomena than what its' title suggest. Needless to say, apart from heaviness of work, there are a lot of other sources of disutility of work, like working conditions, ecological environment and similar. Of course, in these particular cases wages of non-pleasurable occupations should be larger than wages of alternative less difficult occupations. Mj can and should, in that case, be treated as "bad" or "discommodity". A difference between wages of more unpleasant and less unpleasant works presents equalizing differences in the sense explained by Rosen (1987). Note, however, that in our case wage differences can compensate not only for current differences in disutility of work, but also for differences in expected stream of disutility, the second part of expressions (20) and (26) that may be caused by current work. The expected stream of disutility may, for example, take a form of deteriorated quality of life resulting from health problems induced by inadequate working conditions.

There are probably a lot of other interesting cases where proposed models can be used. We mentioned just these that seem to be most interesting and easiest to understand.

References

[1] Abbing, H. (2004). About subsidies income and numbers of artists. 13th ACEI Conference in Chicago.

[2] Adler, M. (1985). Stardom and talent. *American Economic Review,* 75(1): 208–12.

[3] Adler, M. (2006). Stardom and Talent, Volume 1 of *Handbook of the Economics of Art and Culture*, Chapter 25 Elsevier: 895–906.

[4] Alper, N. and Wassall, G. H. (2006). Artists' careers and their labor markets. Volume 1 of *Handbook of the Economics of Art and Culture*, Chapter 23 Elsevier: 813–864.

[5] Angner, E. and Loewenstein G. F. (2012). Behavioral economics. In *Handbook of the Philosophy of Science: Philosophy of Economic* Elsevier: 641–690.

[6] Becker, G. S. (1965). A theory of the allocation of time. *NBER* 75(299): 493–517.

[7] Becker, G. S. (1991). *A Treatise on the Family.* Cambridge, Mass.: Harvard University Press.

[8] Becker, G. S. (1993). The economic way of looking at life. *John M. Olin Law and Economic Working Paper* No. 12 (2nd Series).

[9] Becker, G. S., M. Grossman, and Murphy K. M. (1991). Rational addiction and the effect of price on consumption. *American Economic Review,* 81(2): 237–41.

[10] Bryant, W. D. and D. Throsby (2006). Creativity and the Behavior of Artists, Volume 1 of *Handbook of the Economics of Art and Culture*, Chapter 16 Elsevier: 507–529.

[11] Casacuberta, C. and Gandelman, N. (2012). Multiple job holding: the artist's labor supply approach. *Applied Economics*, Taylor and Francis Journals 44(3): 323–337.

[12] Caserta, M. and Cuccia, T. (2001, August). The supply of arts labor: Towards a dynamic approach. *Journal of Cultural Economics,* 25(3): 185–201.

[13] Galor, O. and Weil, D. N. (1993). The gender gap, fertility, and growth. *NBER Working Papers* 4550.

[14] Ghez, G. and Becker, G. S. (1975). A theory of the allocation of time and goods over the life cycle. In *The Allocation of Time and Goods over the Life Cycle, NBER:* 1–45.

[15] Grossman, M. (1971). The economics of joint production in the household. Report 7145, Center Math. Stud. Bus. Econ., University of Chicago, mimeo.

[16] Heilbrun, J. and Gray, C. (2004). *The Economics of Art and Culture* (2nd Edition). Cambridge University Press.

[17] Jeffri J., Greenblatt, R. F. and Mary, G. (1991). The artists training and career project: Painters. *Research Center for Arts and Culture*, Columbia University, New York.

[18] Jozefowicz, J.J., Kelley, J., and Brewer, S. M. (2008). New release: An empirical analysis of vhs/dvd rental success. *Atlantic Economic Journal,* 36(2): 139–151.

[19] Krusell, P., Ohanian, L. E., Ríos-Rull, J-V, and Violante, G. L. (2000). Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica* 68(5): 1029–1054.

[20] Mckenzie, J. (2010). How do theatrical box office revenues affect dvd retail sales? Australian empirical evidence. *Journal of Cultural Economics,* 34(3): 159–179.

[21] Menger, P.M. (2002). Are there to many artists? The 'excess supply' issue: a measurement puzzle, an increasingly flexibility driven functional requirement and an unavoidable mismatch effect in creative activities. International Symposium on Culture Statistics, Montreal.

[22] Menger, P.M. (2006). Artistic Labor Markets: Contingent Work, Excess Supply, and Occupational Risk Management: 766–811. *Handbook of the Economics of Art and Culture.* Elsevier, Amsterdam.

[23] Merijn, R. and Madden, C. (2000). Living art: Artists between making art and making the living. *Australian Bulletin of Labor,* 26(4): 325–354.

[24] Michael, R.T. and Becker, G.S. (1973). On the New Theory of Consumer Behavior. *The Swedish Journal of Economics,* 75: 378–396.

[25] Nelson, R., Reid, C. and Gilmore, O. (2007). An analysis of the out-of-market gap for dvds in the U.S. *Review of Industrial Organization,* 30(4): 303–323.

[26] Pitt, I. (2010). Superstar effects on royalty income in a performing rights organization. *Journal of Cultural Economics.,* 34(3): 219–236.

[27] Rosen, S. (1981, December). The economics of superstars. *American Economic Review,* 71(5): 845–58.

[28] Rosen, S. (1987). The theory of equalizing differences. In Ashenfelter, O. and Layard, R. (Eds.), *Handbook of Labor Economics*, Volume 1, Chapter 12 Elsevier: 641–692.

[29] Santos, F.P. (1976). Risk, uncertainty, and the performing artist. *The Economics of the Arts*, London: Martin Robertson und Company: 243–259.

[30] Stigler, G.J. and Becker, G.S. (1977). De gustibus non est disputandum. *American Economic Review,* 67(2): 76–90.

[31] Throsby, D. (1992). Artists as workers. In Towse, R. and Khakee, A. (Eds.), *Cultural Economics*, pp. 201–208. Springer Berlin Heidelberg.

[32] Throsby, D. (1994a). The production and consumption of the arts: A view of cultural economics. *Journal of Economic Literature,* 32(1): 1–29.

[33] Throsby, D. (1994b). A work-preference model of artist behavior. In Peacock, A. and Rizzo, I. (Eds.), *Cultural Economics and Cultural Policies,* Springer: 69–80. The Netherlands.

[34] Throsby, D. (2007). Preferred work patterns of creative artists. *Journal of Economics and Finance,* 31(3): 395–402.

[35] Towse, R. (2006). Human capital and artists' labor markets. Volume 1 of *Handbook of the Economics of Art and Culture*, Chapter 24 Elsevier: 865–894.

[36] Walls, W.D. (2009a). Screen wars, star wars, and sequels. *Empirical Economics,* 37(2): 447–461.

[37] Walls, W.D. (2009b). Superstars and heavy tails in pre-recorded entertainment: Empirical analysis of the market for dvds. *Working Papers 2009-20, Department of Economics, University of Calgary.*

APPENDIXES

6.1. A dynamic model with arts labor hired

The artist's decision making process is given by the following expression

$$Uf = \sum_{j=1}^{n} \left(\frac{1}{1+\rho} \right)^j U\left[M_j, Z_j \right]$$

which should be maximized under the following full income constraint

$$\sum_{j=1}^{n} \left(\frac{1}{1+r} \right)^j \left[p^Z X_j^Z + p^M X_j^M + w^n L_j^Z + w^n L_j^M \right] = \sum_{j=1}^{n} \left(\frac{1}{1+r} \right)^j \left[w_j^M L_j^M + w^n L_j + b_j \right]$$

such that $M_j = M\left[X_j^M, L_j^M, H_j \right]$, $Z_j = Z\left[X_j^Z, L_j^Z \right]$, $H_j = h\left[M_{j-1}, M_{j-2}, M_{j-3}, ..., E_j \right]$ and $w_j^M = w^M\left(H_j \right)$.

In order to solve the problem we form the following Lagrange

$$L = \sum_{j=1}^{n} \left(\frac{1}{1+\rho} \right)^j U\left[M_j, Z_j \right]$$

$$-\lambda \sum_{j=1}^{n} \left(\frac{1}{1+r} \right)^j \left[p^Z X_j^Z + p^M X_j^M + w^n L_j^Z + w^n L_j^M - w_j^M L_j^M - w^n L_j - b_j \right]$$

Since X_j^Z and L_j^Z are here derived from the expression $Z_j = Z\left[X_j^Z, L_j^Z \right]$, it follows that X_j^Z and L_j^Z are both the function of Z_j. On the other hand, since X_j^M and L_j^M are both derived from the expression $M_j = M\left[X_j^M, L_j^M, H_j \right]$, it follows that X_j^M and L_j^M are both the function of M_j, M_{j-1}, M_{j-2} and all other previous M. As we already know w_j^M is the function of H_j, which is, in turn, according to the expression of H_j, also the function of all previous M.

The solution of the problem is straightforward. The first order condition requires that partial derivatives of Lagrange with respect to M_j and Z_j be equal to zero. For the sake of simplicity, the second order condition will be skipped in the following consideration[43]. Using the described procedure we get the following solution for a commodity Z_j By the first order condition for Z_j we have

$$\frac{\partial L}{\partial Z_j} = \left(\frac{1}{1+\rho} \right)^j \frac{\partial U}{\partial Z_j} - \lambda \left(\frac{1}{1+r} \right)^j \left[p^Z \frac{dX_j^Z}{dZ_j} + w^n \frac{dL_j^Z}{dZ_j} \right] = 0$$

Hence, we get the following solution for a commodity Z_j

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[43] The second order condition requires that the second partial derivatives of the utility function with respect to M_j and Z_j be negative.

$$\frac{\partial U}{\partial Z_j} = \lambda \left(\frac{1+\rho}{1+r}\right)^j \left[p^Z \frac{dX_j^Z}{dZ_j} + w^n \frac{dL_j^Z}{dZ_j} \right] = \lambda \left(\frac{1+\rho}{1+r}\right)^j \pi_{Z_j}$$

where $\pi_{Z_j} = p^Z \dfrac{dX_j^Z}{dZ_j} + w^n \dfrac{dL_j^Z}{dZ_j}$ presents the shadow price of a commodity Z_j.

By the first order condition for M_j we have

$$\frac{\partial \mathbf{L}}{\partial M_j} = \left(\frac{1}{1+\rho}\right)^j \frac{\partial U}{\partial M_j} - \lambda \left(\frac{1}{1+r}\right)^j p^M \frac{\partial X_j^M}{\partial M_j} - \lambda \left(\frac{1}{1+r}\right)^j w^n \frac{\partial L_j^M}{\partial M_j} + \lambda \left(\frac{1}{1+r}\right)^j w_j^M \frac{\partial L_j^M}{\partial M_j}$$

$$- \lambda \sum_{v=j+1}^{n} \left(\frac{1}{1+r}\right)^v p^M \frac{dX_v^M}{dM_j} - \lambda \sum_{v=j+1}^{n} \left(\frac{1}{1+r}\right)^v w^n \frac{dL_v^M}{dM_j} + \lambda \sum_{v=j+1}^{n} \left(\frac{1}{1+r}\right)^v w_v^M \frac{dL_v^M}{dM_j}$$

$$+ \lambda \sum_{v=j+1}^{n} \left(\frac{1}{1+r}\right)^v L_v^M \frac{dw_v^M}{dM_j} = 0$$

Hence, we get the following solution for M_j

$$\frac{\partial U}{\partial M_j} = \lambda \left(\frac{1+\rho}{1+r}\right)^j \left[\frac{p^M}{\dfrac{\partial M_j}{\partial X_j^M}} + \frac{w^n - w_j^M}{\dfrac{\partial M_j}{\partial L_j^M}} \right]$$

$$+ \sum_{v=j+1}^{n} \frac{(1+\rho)^j}{(1+r)^v} \left[\lambda p^M \frac{dX_v^M}{dM_j} + \lambda \left(w^n - w_v^M\right) \frac{dL_v^M}{dM_j} - \lambda L_v^M \frac{dw_v^M}{dM_j} \right]$$

In order to get more meaningful result we need to make some further transformations. We will use an alternative way of solving the above decision making problem. We first substitute production function M_j and Z_j in utility function U_f. This utility function is supposed to be maximized subject to the full income constraint. Lagrange now takes the following form

$$\mathbf{V} = \sum_{j=1}^{n} \left(\frac{1}{1+\rho}\right)^j U\left[M\left[X_j^M, L_j^M, H_j \right], Z\left[X_j^Z, L_j^Z \right] \right]$$

$$- \lambda \sum_{j=1}^{n} \left(\frac{1}{1+r}\right)^j \left[p^Z X_j^Z + p^M X_j^M + w^n L_j^Z + w^n L_j^M - w_j^M L_j^M - w^n L_j - b_j \right]$$

By solving for the first order condition, that is by equating partial derivatives of this Lagrange with respect to $X_j^Z, L_j^Z, X_j^M, L_j^M$ and H_j to zero, we get a set of equations from which we can derive expressions

$$\lambda p^M = \left(\frac{1+r}{1+\rho}\right)^j \frac{\partial U}{\partial M_j}\frac{\partial M_j}{\partial X_j^M}$$

and

$$\lambda\left(w^n - w_j^M\right) = \left(\frac{1+r}{1+\rho}\right)^j \frac{\partial U}{\partial M_j}\frac{\partial M_j}{\partial L_j^M}$$

These expressions are valid for every j and, therefore, for every v and that is exactly what we need for our further transformations.

By substituting this in the last part of the solution for M_j we get

$$\frac{\partial U}{\partial M_j} = \lambda\left(\frac{1+\rho}{1+r}\right)^j \left[\frac{p^M}{\frac{\partial M_j}{\partial X_j^M}} + \frac{w^n - w_j^M}{\frac{\partial M_j}{\partial L_j^M}}\right] - \lambda\sum_{v=j+1}^{n}\frac{(1+\rho)^j}{(1+r)^v}L_v^M\frac{dw_v^M}{dM_j}$$

$$+\sum_{v=j+1}^{n}\frac{(1+\rho)^j}{(1+r)^v}\frac{\partial U}{\partial M_v}\left[\frac{\partial M_v}{\partial X_v^M}\frac{dX_v^M}{dM_j} + \frac{\partial M_v}{\partial L_v^M}\frac{dL_v^M}{dM_j}\right]$$

Since $\dfrac{dM_v}{dM_j} = 0$ it is obvious that $\dfrac{\partial M_v}{\partial H_v}\dfrac{dH_v}{dM_j} + \dfrac{\partial M_v}{\partial L_v^M}\dfrac{dL_v^M}{dM_j} + \dfrac{\partial M_v}{\partial X_v^M}\dfrac{dX_v^M}{dM_j} = \dfrac{dM_v}{dM_j} = 0$

Hence,

$$\frac{\partial M_v}{\partial L_v^M}\frac{dL_v^M}{dM_j} + \frac{\partial M_v}{\partial X_v^M}\frac{dX_v^M}{dM_j} = -\frac{\partial M_v}{\partial H_v}\frac{dH_v}{dM_j}$$

By substituting this result in the solution for M_j, we finally get

$$\frac{\partial U}{\partial M_j} = \lambda\left(\frac{1+\rho}{1+r}\right)^j \left[\frac{p^M}{\frac{\partial M_j}{\partial X_j^M}} + \frac{w^n - w_j^M}{\frac{\partial M_j}{\partial L_j^M}}\right] - \lambda\sum_{v=j+1}^{n}\frac{(1+\rho)^j}{(1+r)^v}L_v^M\frac{dw_v^M}{dM_j}$$

$$-\sum_{v=j+1}^{n}\frac{(1+\rho)^j}{(1+r)^v}\frac{\partial U}{\partial M_v}\frac{\partial M_v}{\partial H_v}\frac{dH_v}{dM_j}$$

By rearranging it we get an equally useful expression

$$\left[p^M\frac{\partial X_j^M}{\partial M_j} + w^n\frac{\partial L_j^M}{\partial M_j}\right] = \frac{1}{\lambda}\left(\frac{1+r}{1+\rho}\right)^j\frac{\partial U}{\partial M_j} + \frac{1}{\lambda}\sum_{v=j+1}^{n}\frac{(1+r)^j}{(1+\rho)^v}\frac{\partial U}{\partial M_v}\frac{\partial M_v}{\partial H_v}\frac{dH_v}{dM_j}$$

$$+w_j^M\frac{\partial L_j^M}{\partial M_j} + \sum_{v=j+1}^{n}\frac{(1+r)^j}{(1+r)^v}L_v^M\frac{dw_v^M}{dM_j}$$

6.2. A Dynamic Model with Arts Products Sold

The artist's decision making process is given by the following expression

$$Uf = \sum_{j=1}^{n} \left(\frac{1}{1+\rho} \right)^{j} U\left[M_{j}, Z_{j} \right]$$

which should be maximized under the following full income constraint

$$\sum_{j=1}^{n} \left(\frac{1}{1+r} \right)^{j} \left[p^{Z} X_{j}^{Z} + p^{M} X_{j}^{M} + w^{n} L_{j}^{Z} + w^{n} L_{j}^{M} \right] = \sum_{j=1}^{n} \left(\frac{1}{1+r} \right)^{j} \left[p^{A} M_{j} + w^{n} L_{j} + b_{j} \right]$$

such that $M_{j} = M\left[X_{j}^{M}, L_{j}^{M}, H_{j} \right]$, $Z_{j} = Z\left[X_{j}^{Z}, L_{j}^{Z} \right]$, $H_{j} = h\left[M_{j-1}, M_{j-2}, M_{j-3}, ..., E_{j} \right]$ and $w_{j}^{M} = w^{M}\left(H_{j} \right)$.

In order to solve the problem we form the following Lagrange

$$\text{L} = \sum_{j=1}^{n} \left(\frac{1}{1+\rho} \right)^{j} U\left[M_{j}, Z_{j} \right]$$

$$-\lambda \sum_{j=1}^{n} \left(\frac{1}{1+r} \right)^{j} \left[p^{Z} X_{j}^{Z} + p^{M} X_{j}^{M} + w^{n} L_{j}^{Z} + w^{n} L_{j}^{M} - p^{A} M_{j} - w^{n} L_{j} - b_{j} \right]$$

The same as before, since X_{j}^{Z} and L_{j}^{Z} are here derived from the expression $Z_{j} = Z\left[X_{j}^{Z}, L_{j}^{Z} \right]$, it follows that X_{j}^{Z} and L_{j}^{Z} are both the function of Z_{j}. On the other hand, since X_{j}^{M} and L_{j}^{M} are both derived from the expression $M_{j} = M\left[X_{j}^{M}, L_{j}^{M}, H_{j} \right]$, it follows that X_{j}^{M} and L_{j}^{M} are both a function of M_{j}, M_{j-1}, M_{j-2} and all other previous M.

The first order condition for the solution of the problem requires that partial derivatives of Lagrange with respect to M_{j} and Z_{j} be equal to zero. For the sake of simplicity, we again skip consideration of the second order condition. Using the described procedure we get the solution for commodity Z_{j} which is exactly the same as the one obtained previously in expression (18). As we know, the marginal utility of commodity Z_{j} should be equal to the marginal cost of all inputs involved in production of that commodity, which is equal to the shadow price of commodity Z_{j}.

By the first order condition for M_{j} we have

$$\frac{\partial \text{L}}{\partial M_{j}} = \left(\frac{1}{1+\rho} \right)^{j} \frac{\partial U}{\partial M_{j}} - \lambda \left(\frac{1}{1+r} \right)^{j} p^{M} \frac{\partial X_{j}^{M}}{\partial M_{j}} - \lambda \left(\frac{1}{1+r} \right)^{j} w^{n} \frac{\partial L_{j}^{M}}{\partial M_{j}} + \lambda \left(\frac{1}{1+r} \right)^{j} p^{A}$$

$$-\lambda \sum_{v=j+1}^{n} \left(\frac{1}{1+r} \right)^{v} p^{M} \frac{dX_{v}^{M}}{dM_{j}} - \lambda \sum_{v=j+1}^{n} \left(\frac{1}{1+r} \right)^{v} w^{n} \frac{dL_{v}^{M}}{dM_{j}} = 0$$

Hence, we get the following solution for M_{j}

$$\frac{\partial U}{\partial M_{j}} = \lambda \left(\frac{1+\rho}{1+r} \right)^{j} \left[p^{M} \frac{\partial X_{j}^{M}}{\partial M_{j}} + w^{n} \frac{\partial L_{j}^{M}}{\partial M_{j}} - p^{A} \right] + \sum_{v=j+1}^{n} \frac{(1+\rho)^{j}}{(1+r)^{v}} \left[\lambda p^{M} \frac{dX_{v}^{M}}{dM_{j}} + \lambda w^{n} \frac{dL_{v}^{M}}{dM_{j}} \right]$$

In order to make this solution more understandable we will further transform it. We will use, again, an alternative way of solving the above decision making problem. We first substitute the production function M_j and Z_j in utility function U_f. This utility function is supposed to be maximized subject to the full income constraint. Lagrange now takes a new form

$$V = \sum_{j=1}^{n} \left(\frac{1}{1+\rho}\right)^j U\left[M\left[X_j^M, L_j^M, H_j\right], Z\left[X_j^Z, L_j^Z\right]\right]$$

$$-\lambda \sum_{j=1}^{n} \left(\frac{1}{1+r}\right)^j \left[p^Z X_j^Z + p^M X_j^M + w^n L_j^Z + w^n L_j^M - p^A M_j - w^n L_j - b_j\right]$$

By solving for the first order condition, that is by equating partial derivatives of this Lagrange with respect to $X_j^Z, L_j^Z, X_j^M, L_j^M$ and H_j to zero, we get a set of equations from which we can derive expressions

$$\lambda p^M = \left(\frac{1+r}{1+\rho}\right)^j \frac{\partial U}{\partial M_j} \frac{\partial M_j}{\partial X_j^M} + \lambda p^A \frac{\partial M_j}{\partial X_j^M}$$

and

$$\lambda w^n = \left(\frac{1+r}{1+\rho}\right)^j \frac{\partial U}{\partial M_j} \frac{\partial M_j}{\partial L_j^M} + \lambda p^A \frac{\partial M_j}{\partial L_j^M}$$

These expressions are valid for every j and, therefore, for every v and that is exactly what we need for our further transformations.

By substituting this in the last part of the solution for M_j we get

$$\frac{\partial U}{\partial M_j} = \lambda \left(\frac{1+\rho}{1+r}\right)^j \left[p^M \frac{\partial X_j^M}{\partial M_j} + w^n \frac{\partial L_j^M}{\partial M_j} - p^A\right]$$

$$+ \sum_{v=j+1}^{n} \frac{(1+\rho)^j}{(1+\rho)^v} \frac{\partial U}{\partial M_v} \left[\frac{\partial M_v}{\partial X_v^M} \frac{dX_v^M}{dM_j} + \frac{\partial M_v}{\partial L_v^M} \frac{dL_v^M}{dM_j}\right]$$

$$+ \lambda p^A \sum_{v=j+1}^{n} \frac{(1+\rho)^j}{(1+r)^v} \left[\frac{\partial M_v}{\partial X_v^M} \frac{dX_v^M}{dM_j} + \frac{\partial M_v}{\partial L_v^M} \frac{dL_v^M}{dM_j}\right]$$

Since expressions in the second and third brackets are the same and equal to:

$$\frac{\partial M_v}{\partial L_v^M} \frac{dL_v^M}{dM_j} + \frac{\partial M_v}{\partial X_v^M} \frac{dX_v^M}{dM_j} = -\frac{\partial M_v}{\partial H_v} \frac{dH_v}{dM_j}$$

By substituting this result in the solution for M_j, we finally get

$$\frac{\partial U}{\partial M_j} = \lambda \left(\frac{1+\rho}{1+r}\right)^j \left[\frac{p^M}{\frac{\partial M_j}{\partial X_j^M}} + \frac{w^n - p^A \frac{\partial M_j}{\partial L_j^M}}{\frac{\partial M_j}{\partial L_j^M}}\right]$$
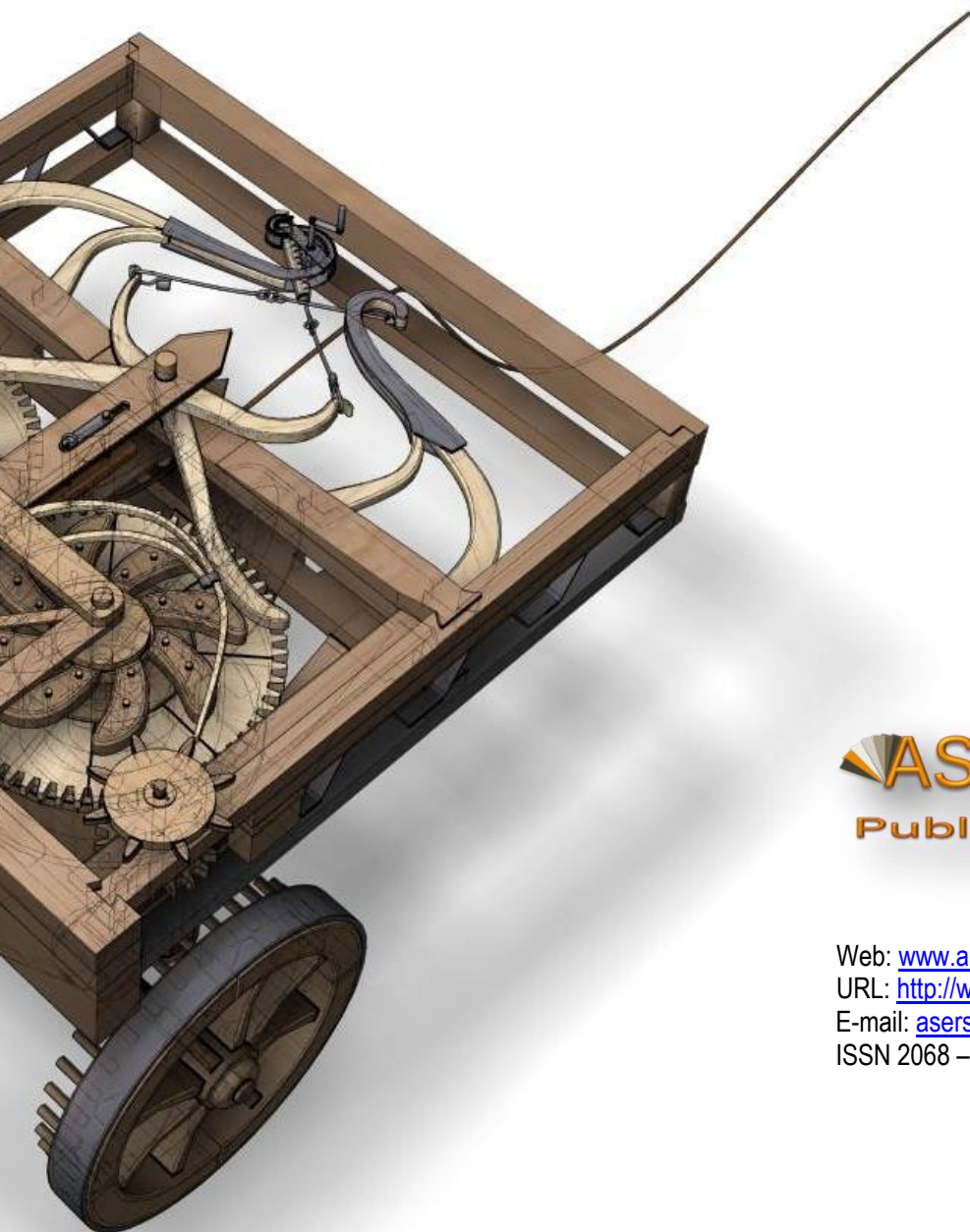
$$-\lambda p^{A} \sum_{v=j+1}^{n} \frac{(1+\rho)^{j}}{(1+r)^{v}} \frac{\partial M_{v}}{\partial H_{v}} \frac{dH_{v}}{dM_{j}} - \sum_{v=j+1}^{n} \frac{(1+\rho)^{j}}{(1+\rho)^{v}} \frac{\partial U}{\partial M_{v}} \frac{\partial M_{v}}{\partial H_{v}} \frac{dH_{v}}{dM_{j}}$$

By rearranging it we get the following useful expression

$$\left[p^{M} \frac{\partial X_{j}^{M}}{\partial M_{j}} + w^{n} \frac{\partial L_{j}^{M}}{\partial M_{j}} \right] = \frac{1}{\lambda} \left(\frac{1+r}{1+\rho} \right)^{j} \frac{\partial U}{\partial M_{j}} + \frac{1}{\lambda} \sum_{v=j+1}^{n} \frac{(1+r)^{j}}{(1+\rho)^{v}} \frac{\partial U}{\partial M_{v}} \frac{\partial M_{v}}{\partial H_{v}} \frac{dH_{v}}{dM_{j}}$$

$$+ p^{A} + p^{A} \sum_{v=j+1}^{n} \frac{(1+r)^{j}}{(1+r)^{v}} \frac{\partial M_{v}}{\partial H_{v}} \frac{dH_{v}}{dM_{j}}$$

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